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The terrestrial Planet V hypothesis as the mechanism for the origin of the late heavy bombardment

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ABSTRACT

In this study we attempt to model, with numerical simulations, the scenario for the origin of the late heavy bombardment (LHB) proposed in Chambers (2007, Icarus, 189, 386). Chambers suggested that the orbit of a fictitious sub-Mars sized embryo outside Mars became unstable and crossed a part of the asteroid belt until it was removed through ejection or collision. Chambers demonstrated that this kind of evolution can occur at the LHB time, but he did not check whether enough material could be liberated from the belt to be consistent with the magnitude of the LHB. We do that here. In addition to the asteroid belt, we also consider putative belts of small objects located between the orbits of the Earth and Venus (EV belt) or between the Earth and Mars (EM belt). We find that delivering enough material to the Moon from the asteroid belt requires at least a 5 lunar mass embryo crossing the entire asteroid belt up to 3.3 AU for longer than 300 Myr on a low-inclination orbit. We do not witness this to occur in over a hundred simulations. An alternative approach, which relies only on decimating the inner belt (<2.5 AU), requires that the inner belt originally contained 4 to 13 times more mass than the original outer belt. Regarding the putative asteroid belts in between the terrestrial planets, this scenario could be consistent with the LHB if EV belt contained a substantial population of small bodies, with the density of objects in orbital space larger than that of the current main asteroid belt, while the EM belt was as good as empty.

Key words. planets and satellites: dynamical evolution and stability

1. Introduction

The last impact basins on the Moon formed about 3.8 Gyr ago, that is about 600 Myr after the formation of the Moon itself. It has been intensely debated whether these late basins are just the last big impacts that occurred during a bombardment declining in intensity since the time of the formation of the Moon (Hartmann 1975; Neukum & Ivanov 1996; Hartmann et al. 2007), or if these basins formed during a sudden surge in the impact rate, called the late heavy bombardment (LHB; Tera et al. 1974; Ryder 1990; Kring & Cohen 2002; Bottke et al. 2007). Although there is still no unanimous consensus, most evidence now supports the latter hypothesis (see Hartmann et al. 2000; and Chapman et al. 2007 for reviews).

A dynamical scenario for the origin of the LHB was proposed by Chambers (2007). He suggests that the LHB is related to an instability of the terrestrial planet system, which is initially composed of five planets. Chambers (2007) showed that a system with a fifth planet of sub-martian mass, Planet V, originally located between Mars and the inner edge of the asteroid belt, could have remained stable for long time (hundreds of millions of years) and eventually become unstable. Afterwards, the evolution of the rogue planet would have proceeded through encounters with Mars, which caused Planet V's orbit to evolve on a timescale of the order of 100 Myr. Eventually, the rogue planet could have become Earth-crosser and was removed from the system either by collision with a planet, collision with the Sun or ejection by Jupiter. Chambers proposed that, during its high eccentricity phase, the rogue planet could have crossed the asteroid belt and dislodged enough asteroids to cause the LHB on the terrestrial planets, but he did not investigate this aspect quantitatively with his numerical simulations.

The purpose of this paper is to evaluate this last possibility. In what follows, we shall use both the term "Planet V" and "embryo" to designate the same object (i.e. the fifth terrestrial planet). In Sect. 2 we outline our methods and initial conditions for the numerical simulations. In Sect. 3 we present the results of our numerical simulations both for the asteroid belt and for putative belts of small objects located between Venus and the Earth or the Earth and Mars. Section 4 states our conclusions.

2. Methods and set up

This study consists of a large sample of numerical simulations. All simulations contained the following massive bodies: the Sun, the terrestrial planets Venus, Earth and Mars, the fictitious Planet V, and the gas giants Jupiter and Saturn. Mercury, Uranus and Neptune were not included. Several simulations also included either the asteroid belt or putative asteroid belts between Venus and Earth, and Earth and Mars (EV and EM belts; Evans & Tabachnik 1999). The simulations were performed using the symplectic integrator SyMBA (Duncan et al. 1998), which is able to model planet-planet scattering. We used a time step of 0.03 yr, unless specified otherwise. Bodies were removed when a collision occured with a planet, the Sun, or when they were farther than 20 AU from the Sun. In SyMBA the radius of the Sun was kept at its current value but we checked for close encounters with the Sun. All simulations were performed on the CRIMSON Beowulf cluster at the Observatoire de la Côte d'Azur. The typical simulation time when accounting for the asteroids is 4–6 months. Below we discuss the initial conditions for planets and small body populations.

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2.1. Simulations of the 5-planet terrestrial system

In order to characterise the variety of the possible outcomes of the fictitious 5-planet terrestrial system, we performed two sets of 132 simulations featuring only the planets Venus, Earth, Mars, Jupiter, Saturn and Planet V. The latter had a mass of 1, 2.5, 5 and 10 lunar masses. A total of 33 runs were performed for each value of the mass, with each simulation running for up to 500 Myr; when the embryo (Planet V) was lost the simulation was stopped. We used the current orbits for the real planets, whose initial conditions were taken from the IMCCE webpages. In the first set of simulations the initial conditions for the embryo correspond to orbits with orbital elements chosen randomly in the following intervals: semi-major axis $a \in [1.7, 1.75]$ AU, eccentricity $e \in [0, 0.05]$, inclination $i \in [0, 5°]$ (with uniform distribution in $\sin i$), longitude of ascending node Ω, argument of pericentre ω and mean anomaly $M \in [0, 360°)$. In the second set of simulations the embryo had $q = 1.65$ AU and semi-major axis $a \in [1.90, 1.95]$ AU ($e \sim 0.14$). These initial conditions resemble Chambers' (2007) values, but were modified in either semi-major axis or eccentricity to ensure that the embryo encountered Mars near the beginning of the simulation. This is done because we are not interested in the possibility that the system can remain stable for long time before becoming unstable, already demonstrated in Chambers (2007); instead we are interested only in the evolution of the embryo after the instability. Thus, choosing initial conditions that provide an early instability is an effective way for us to save computing time.

2.2. Asteroid belt

It is unlikely that the asteroid belt at the time of the LHB had a dynamical state very different from the current one (Petit et al. 2001; Levison et al. 2001; Morbidelli et al. 2010). Thus, to construct a plausible proxy of the orbital distibution of the asteroid belt at that epoch, we initially constructed a uniform population of test particles with semi-major axes from 1.8 AU to 4.5 AU, inclinations up to 20° and eccentricities between 0 and 0.3, such that the pericentre distance $q > 1.8$ AU and the apocentre $Q < 4.5$ AU (Morbidelli et al. 2010). We evolved this population for 100 Myr, under the gravitational influence of the Sun, Jupiter and Saturn. Particles were removed when they entered the Hill sphere of a planet, came closer than 1.8 AU to the Sun or when they were farther than 20 AU from the Sun. The surviving particles constitute our asteroid belt model. The adopted timescale of 100 Myr is shorter than the age of the LHB relative to planet formation (600 Myr), but the fraction of asteroids expected to escape between 100 and 600 Myr is only approximately 10% of the total (Minton & Malhotra 2010) and the exclusion of this population would barely affect the results.

In some of the simulations we took into account the gravitational effects of the asteroids on the planets. For this purpose, the asteroid belt was assumed to have a total mass of approximately 12 lunar masses distributed in 3325 objects. This value was derived from an estimate of the mass that the asteroid belt should have had in order to be the dominant source of the LHB. This estimate is achieved as follows. From the distribution of craters and basins on the Moon, it is estimated that about 3×10^{18} to 10^{19} kg of projectiles hit the Moon during the LHB (Levison et al. 2001); for asteroids escaping from all over the asteroid belt, the typical collision probability with the Moon is approximately 10^{-4} (Morbidelli et al. 2002, 2010). Hence the mass of the belt at the LHB time had to be from 3×10^{22} to 10^{23} kg. For comparison, the current asteroid belt contains approximately 3.6×10^{21} kg (Krasinsky et al. 2002), so that Planet V needs to remove between 90% to 95% of the pre-LHB belt in no more than 300 Myr (the estimated maximal duration of the LHB spike).

In contrast, if the LHB had been caused entirely by asteroids from the inner belt, the total mass of the inner belt should have been between 2×10^{22} to 5×10^{22} kg, which is less than that of the full asteroid belt estimated above because the individual collision probability of objects escaping from the inner belt with the Moon is roughly 2 times larger (Morbidelli et al. 2002, 2010). However, we estimate that the inner belt contains currently just ~10% of the total mass of the main belt[1]. Thus, Planet V should have been able to remove more than 98% of the original inner belt asteroid population.

We will return to these removal fractions in Sect. 3.2 when analysing the results of our numerical simulations.

2.3. Asteroid belts in between the terrestrial planets

We performed additional sets of simulations where the asteroids are situated in putative belts between Venus and Earth (EV belt) as well as Earth and Mars (EM belt). These belts are stable for long times, comparable to or even longer than the LHB time (Evans & Tabachnik 1999). Thus, it is a priori possible that asteroids inhabited these regions at the LHB time and that they were destabilised by the direct and indirect perturbations exerted by the wandering Planet V.

To define plausible initial conditions for these populations, we placed Venus, Earth, Mars, Jupiter and Saturn on their current orbits and added 2000 test particles, with semi-major axes between 0.73 AU and 1.47 AU, with $e < 0.2$ and $i < 10°$. We simulated this system for 100 Myr. Asteroids were removed when they entered the Hill sphere of a planet. The asteroids that remained after 100 Myr were typically close to a mean-motion resonance with the Earth and Venus, or were situated in a stable niche between 1.2 and 1.3 AU of the EM belt. Evans & Tabachnik (1999) reported a similar configuration for long-lived bodies. The remaining bodies were cloned twenty times adding a random change of 10^{-5} to the mean anomaly for a total of 1740 asteroids, with 609 in the EV belt and 1131 in the EM belt. These asteroids make up our orbital distribution models of the putative EV and EM belts. We tested the validity of our cloning procedure by integrating the cloned asteroids and the current planets of the solar system and checking that the decay rate of the cloned particles is not significantly faster than that of their original counterparts.

The EV and EM belts now contain no bodies. It is interesting to compute the characteristic size of an asteroid that can be removed from these regions by the drift in semi major axis induced by the Yarkovsky effect, over the post-LHB 3.8 Gyr term. Studies of the Yarkovsky effect (see Bottke et al. 2006 for a review) show that the typical drift rate is $|\dot{a}| \sim 2 \times 10^{-5}$ AU Myr^{-1} for an object with diameter $D = 5$ km in the inner asteroid belt ($a \sim 2$ AU). For objects with $D \gtrsim 1$ km $\dot{a} \propto D^{-1}$ and $\dot{a} \propto a^{-1/2}$. Thus for an object in the vicinity of the Earth $|\dot{a}| \sim 10^{-4}(1\,\mathrm{km}/D)$ AU Myr^{-1}, although this value is uncertain by at least a factor of two. To survive the Yarkovsky drift over the age of the solar system asteroids have to drift less than 0.2 AU (the approximate width of the EV and EM belts) over

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[1] We achieve this estimate by counting the fraction of the asteroid population with absolute magnitude $H < 10$ that resides in the inner belt and assuming that the differences in albedo between inner and outer main belt asteroids are compensated by the differences in bulk densities.

approximately 4 Gyr. Setting $\Delta t = 4000$ Myr and $\Delta a = 0.2$ AU yields a critical size $D_c = 0.5$ km. Given the uncertainties in the drift rates, we decided to be conservative and set $D_c = 1$ km. This is the maximal size of bodies that could be effectively removed from the EV and EM belts by the Yarkovsky drift over the age of the Solar System. Objects larger than this size would remain today, if they survived the LHB phase. In other words, the instability of the terrestrial planets caused by Planet V should have removed all objects down to kilometer size.

3. Results

In this section the results of our numerical simulations are discussed. This section is divided into three subsections: one discussing the evolution of the planets alone, one discussing the decay of the asteroid belt and the last one discussing the evolution of the fictitious EV and EM belts.

3.1. Planetary evolution

Here we give a brief overview of the results of our simulations of a planetary system made of the planets Venus to Saturn, and of Planet V. No small bodies were included in the simulations.

An example of the typical evolution of the system is depicted in Fig. 1. For the first 80 Myr Planet V has repeated encounters with Mars which cause small stochastic changes in the embryos' eccentricity and semi major axis. The aphelion distance of the embryo always stays below 2.2 AU, so that only the very inner part of the asteroid belt is expected to be affected by this evolution. After 80 Myr the embryo's eccentricity suddenly increases and the embryo starts to cross the Earth's orbit. The evolution in semi-major axis of the embryo is now much more rapid, and the Earth quickly scatters the embryo first inwards and then outwards. After another 10 Myr the embryo's eccentricty is increased even further and it ends up crossing the orbits of Venus and Jupiter. A close encounter with Jupiter ejects it onto an hyperbolic orbit. Chambers (2007) also reported this kind of behaviour (see Fig. 1 in that paper). Unfortunately, this course of events, rather common, should not be very effective in generating the LHB because Planet V does not spend much time on an orbit deeply crossing the asteroid belt.

We compiled statistics of how long an embryo crosses the asteroid belt, which occurs when $Q > 2.1$ AU, and what is its characteristic maximum aphelion distance. The results are summarised in Fig. 2. The top panels pertain to the simulations where the initial embryo orbits have $a \sim 1.7$ AU and $e \sim 0$ while the bottom panels refer to the simulations where the embryo initially had $a \sim 1.9$ AU and $e \sim 0.14$. The left panels show the probability that the embryo spends a given time on an orbit with $Q > 2.1$ AU. As one can see, in 30% to 50% of the simulations, the time spent by the embryo on an orbit with $Q > 2.1$ AU is shorter than 30 Myr, suggesting that the system has difficulty keeping the embryo crossing the asteroid belt for a long time. The mean time the embryo spends on an orbit with $Q > 2.1$ AU is 37 Myr for the initially circular orbits, which coincides well with the median time from the top panel. For the eccentric case this is somewhat longer, but is skewed by the initial conditions since the embryo starts with $Q \sim 2.2$. The results in the left panels cannot be directly compared with the statistics reported in Chambers (2007). In fact, Chambers considered the timespan ranging between the first and the last passages of an embryo through the asteroid belt (see his Table 2 and Fig. 13), whereas here we consider the cumulative time spent by the embryo on an orbit crossing the asteroid belt.

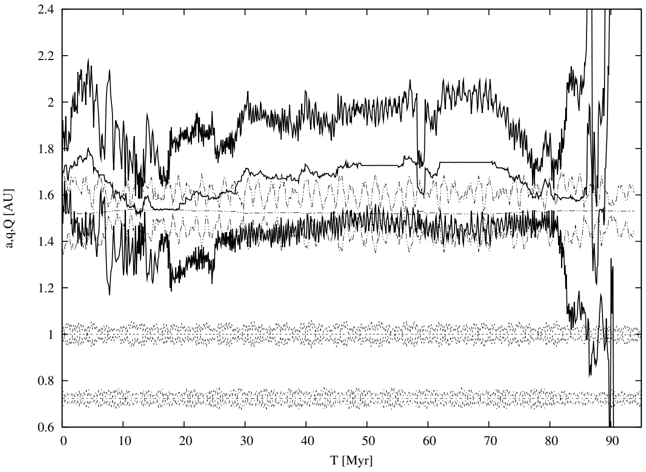

Fig. 1. A fairly typical example of evolution of the terrestrial planet system from our simulations. The semi-major axis, pericentre and apocentre distance of Venus (bottom curves), Earth (curves around 1.0), Mars (dashed lines around 1.5) and the embryo (solid curves) are shown as a function of time. The mass of the embryo is here 1 lunar mass. The embryo is removed by ejection onto hyperbolic orbit after 90 Myr.

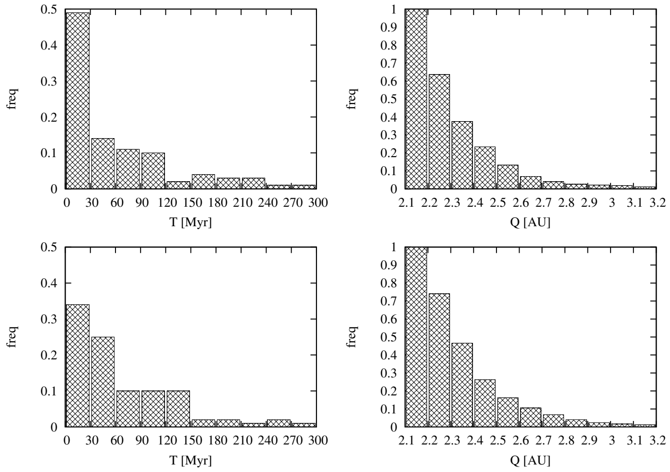

Fig. 2. *Left panels*: probability distribution of the time the embryo spends crossing the asteroid belt. The y-axis reports the fraction of the total number of simulations in which the cumulative time spent by the embryo on an orbit with $Q > 2.1$AU is in each of the time-intervals reported on the x-axis. *Right panels*: probability distribution of aphelion distances for embryos crossing the asteroid belt. Here the y-axis reports the fraction of the total time that the embryo crosses the belt spent on orbits with Q falling in each of the intervals reported on the x-axis. The plotted distribution is the average of the individual distributions of all simulated embryos. These individual distributions have been weighted by the total time the embryo crosses the belt. Panels in the top row correspond to initial embryo orbits with $a \sim 1.7$ AU and $e \sim 0$ while those in the bottom row pertain to orbits with $a \sim 1.9$ AU and $e \sim 0.14$.

The right panels of Fig. 2 depict what fraction of the total time the embryo crosses the asteroid belt is spent on an orbit with a given value of Q. The distributions for both the initially circular and eccentric embryo orbits look essentially the same. A best fit through the data shows that this fraction scales as e^{-Q}. The figure indicates that the embryo spends just 20% of the time that it crosses the asteroid belt on an orbit with $Q > 2.5$ AU and less than 10% of the time on an orbit with $Q > 2.6$ AU. Thus, the time it spends crossing the outer asteroid belt ($Q > 2.5$ AU) is rather short; if it happens at all. We find that the "typical" time

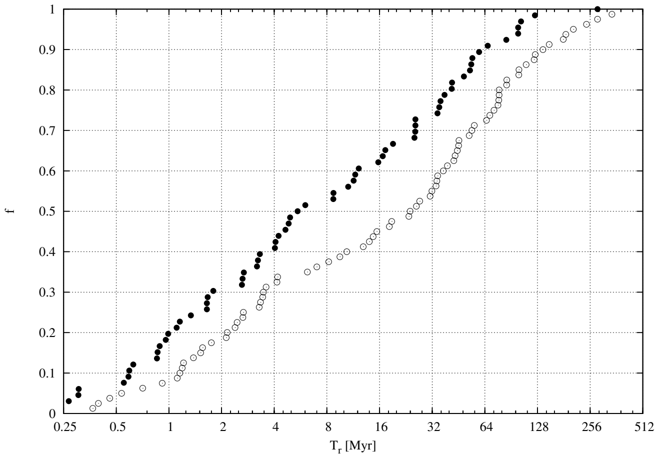

Fig. 3. Cumulative distribution of the dynamical lifetimes of embryos, measured since the time they first reach Earth-crossing (open circles) or Venus-crossing orbits (bullets).

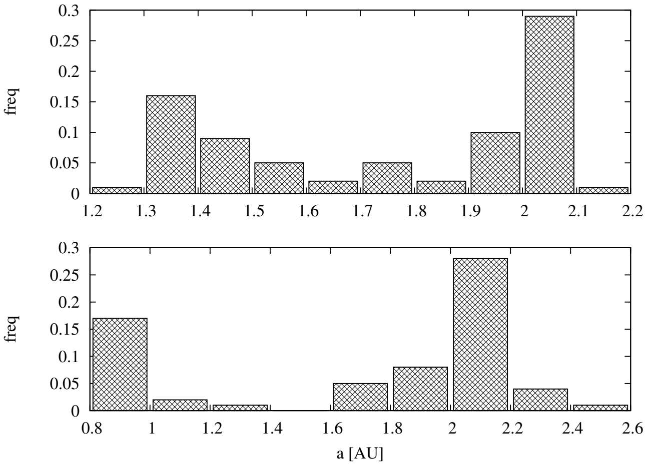

Fig. 4. Histogram of semi-major axis distribution of the embryo when it first reaches an orbit with $q < 1$ AU (*top panel*) and $q < 0.7$ AU (*bottom panel*).

that the embryo spends crossing a part of the belt is 37 Myr. If only 20% of this is spent with $Q > 2.5$ AU implies that the "typical" time it would spend on such an orbit is just 7 Myr; for orbits with Q between 2.2 and 2.3 AU, which is the most common behaviour that we observed, this "typical" time becomes approximately 22 Myr. In Sect. 3.2 we shall discuss what the above results imply for the depletion of the main asteroid belt and for the origin of the LHB from that source.

The reason for which the embryo has difficulty in achieving and sustaining orbits with a large aphelion distance for a long time is the following. When the embryo does cross the full asteroid belt ($Q \sim 3$ AU), it has two kinds of orbit: it either also crosses the orbit of the Earth (or comes close to it) or it does not. In the former case it undergoes close encounters with our planet and therefore it typically has a short dynamical lifetime (~20 Myr; Chambers 2007). In the latter case, which can exist for hundreds of Myr, the embryo must be on an orbit with semi-major axis larger than 2 AU. These orbits, however, are difficult to achieve when starting from the vicinity of Mars. In fact, without suffering close encounters with the Earth, the embryo can evolve from an orbit with $a < 2$ AU to $a > 2$ AU only through close encounters with Mars. However, the random walk in semi-major axis induced by martian encounters has steps of very small amplitude: typically $\Delta(1/a) \ll 10^{-3}$ AU^{-1}. Thus it is unlikely that Mars can scatter the embryo across the ν_6 resonance, which is located at $a \sim 2$ AU and has a width of a few tenths of AU (Morbidelli & Gladman 1998). Instead, in most cases the embryo is temporarily trapped in the ν_6 resonance. The resonance then increases the eccentricity of the embryo's orbit in typically less than 1 Myr, forcing it to become an Earth-crosser or to even collide with the Sun. Thus, the dynamical lifetime of the embryo when crossing the asteroid belt is rather short once again.

For what concerns the fate of the putative asteroid belts between the Earth and Venus and between the Earth and Mars, a quantity of interest is the fraction of simulations in which the embryo becomes Earth or Venus crosser and the time it remains on such crossing orbits. We find that in 80% of cases the embryo becomes Earth crosser and in 66% of cases it becomes Venus crosser. The distribution of the embryo's lifetimes after Earth-crossing and Venus-crossing orbits are achieved is illustrated in Fig. 3. The median lifetime after achieving Earth crossing is ~20 Myr, similar to that characterising Earth-crossing NEAs (Gladman et al. 1997).

In Fig. 4 we plot a histrogram of the semi-major axes of the embryo's orbit when it first crosses the orbits of the Earth (top panel) or Venus (bottom panel). In the top panel there are two peaks in the distribution: one at 1.3 AU and another at 2.0 AU. This distribution suggests that there are two main routes to reach Earth-crossing orbits from the vicinity of Mars. The first, associated with the peak at ~1.3 AU, is a direct transfer from Mars to the Earth by martian encounters. The second, associated with the peak at ~2 AU is an indirect transfer, during which the embryo is first scattered outwards by Mars until it reaches the ν_6 secular resonance, which then increases its eccentricity up to Earth-crossing values. In the bottom panel, we again see two peaks: one around 0.8 AU and the other around 2 AU. The explanation for the peak at 2 AU is the same as above, while the peak at 0.8 AU can be explained by the direct transfer of the embryo from the Earth to Venus due to Earth encounters. Note that the peak around 0.8 AU is rather shallow, suggesting that transfer from Earth to Venus is not very efficient and that instead most Earth-crossing embryos with $a \sim 1.3$ AU are removed by the Earth before having a chance to cross the orbit of Venus.

Given this overview of the various dynamical outcomes we next look at the effects of the embryo on the belts of small bodies, focusing on the a-priori most favourable scenarios in the next subsections.

3.2. The main asteroid belt

From the previous subsection it is clear that, on average, the embryo does not spend a long time on an eccentric orbit crossing the asteroid belt. Thus it is more effective to investigate how long an embryo needs to cross the belt in order to deplete it sufficiently, i.e. 95% (see Sect. 2.1) using idealised simulations, and then compare the results with the time it crosses the belt from the previous subsection. To do so we ran six simulations using SWIFT RMVS3 (Levison & Duncan 1994) in two sets of three. Each simulation ran for 300 Myr with a time step of 0.1 yr. For the first set, the original orbit of Planet V has $q = 1.8$ AU and $Q = 2.5$ AU with $i = 1.5°$, so that it crosses only the inner belt. For the second set we originally placed Planet V on an orbit with $q = 1.9$ AU and $Q = 3.3$ AU so that it crosses the

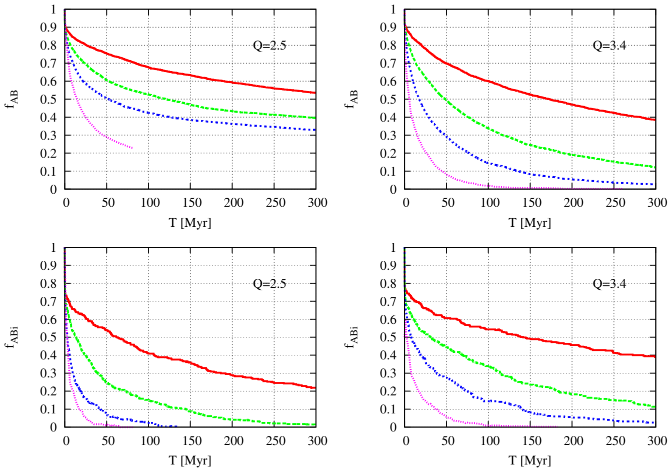

Fig. 5. *Top-right panel*: decay of the whole asteroid belt population in simulations where the embryo has Q = 3.3 AU. The red curves pertain to an embryo with 1 lunar mass, the green curves are for 2.5 lunar masses, the blue curves are for 5 lunar masses and the magenta curves are for the 10 lunar mass (Mars mass) embryo. *Top-left panel*: same as top-right but for an embryo with Q = 2.5 AU. *Bottom panels*: the same as the top panels but for the inner belt only.

whole asteroid belt at low inclination. In all simulations the terrestrial planets were not included so that the orbit of the embryo did not change with time due to close encounters with the former. However, we did include the perturbations from Jupiter of and Saturn. The mass of Planet V differed from one simulation to the other and was equal to 1, 2.5 and 5 lunar masses respectively. Asteroids, which were treated as massless particles, were removed when they came closer than 1.8 AU to the Sun because in reality they would become near Mars crossers and would be eventually removed. We also discarded them when they were farther than 20 AU from the Sun.

The fraction of the asteroid belt that was removed in these simulations is plotted in Fig. 5. The top-left panel plots the decay of the whole belt for the simulations with the embryo having Q = 2.5 AU while the top-right panel depicts the same for an embryo having Q = 3.3 AU. As one can see, the goal of 95% (90%) removal is only achieved by a 5 lunar mass embryo crossing the full belt for approximately 300 Myr (150 Myr). Of course, the decay of the asteroid belt can be sped up by increasing the mass of the embryo. For instance, a 10 lunar mass (Mars mass) embryo would deplete the belt by 90% in just 50 My (see top-right panel of Fig. 5). However, a study similar to that of Sect. 3.1 but with initially circlar terrestrial planets shows that if the embryo is as massive as Mars, the final orbital excitation of the terrestrial planets (as quantified by the Angular Momentum Deficit – see Sect. 3.3.1; and Laskar 1997) is always much larger than its current value. Thus, the existence of a rogue Mars mass embryo appears inconsistent with the current structure of the inner solar system. For this reason we no longer comment on results related to a Mars mass embryo below.

In reality the embryo may remove from the belt a mass comparable to its own mass and thus dynamical friction could play an important role in its dynamics. One may wonder whether dynamical friction could circularise the orbit of the embryo and permanently trap it in the asteroid belt, which would obviously violate observational constraints. To test this possibility we ran simulations with an embryo with q = 1.9 AU and Q = 3.3 AU at low inclination using SyMBA to determine the strength of the dynamical friction. In these simulations the asteroid belt contained once again a total mass of 12 lunar masses divided into 3325 equal-mass bodies. From these simulations we concluded that dynamical friction does not circularise Planet V's orbit. This caused some concern and we repeated the above simulations without Jupiter and Saturn. We found that dynamical friction did indeed decrease the embryo's eccentricity at an initial typical rate $\dot{e}/e \sim -2.5 \times 10^{-8}$ yr^{-1}. The embryo's pericentre increased from 1.9 AU to 2 AU in approximately 3–8 Myr, commensurate with the time scale for secular perturbations of the embryo-asteroid belt system. However, when Jupiter and Saturn are included, the secular decrease in the embryo's eccentricity is no longer observed and the mean value of the embryo's pericentre distance stays constant. We believe that this lack of dynamical friction is caused by the fact that the eccentricity evolution of the asteroids and the embryo are being dominated by Jupiter rather than their mutual interaction. As a result the dynamical friction no longer relies on secular embryo-particle interaction but on two-body scattering events, which operates on a typical time scale of ~1 Gyr. Additionally, the removal of asteroids does not occur because encounters with the embryo increase the asteroids' eccentricity above the limit set by q = 1.8 AU. Instead, scattering off the embryo gives the asteroids some mobility in semi-major axis so that they can fall into a mean-motion resonance with Jupiter. These resonances quickly increase the asteroids' eccentricities and they are removed. Thus Jupiter is doing most of the lifting of the asteroids' eccentricities, and not the embryo, which redices the effect of dynamical friction. The lack of dynamical friction is an important observation, because it shows that an embryo destabilizing a relatively massive asteroid belt would most likely not be permanently trapped in this belt.

Unfortunately, among the many simulations of the evolution of the full planetary system reported in Sect. 3.1, none showed an embryo crossing the whole belt for a timescale of several hundreds of millions of years, as required for the global depletion of the asteroid belt population. The evolution of the cases where the embryo crosses the asteroid belt the longest are illustrated in Fig. 6. As one can see, the embryo crosses the inner portion of the asteroid belt for longer than 300 Myr, but only rarely does it achieve an orbit with $Q \gtrsim 2.5$ AU. This suggests that the inner belt might have been depleted considerably while the outer belt has not. In order to determine whether or not this is a viable possibility we have plotted the decay of the inner belt for the two sets of idealised simulations in the bottom two panels of Fig. 5. The bottom-left panel contains the results for the simulations where the embryo has Q = 2.5 AU while in the bottom-right panel the embryo has Q = 3.3 AU. We have seen in Sect. 2.2 that the inner belt should have contained 2−5 × 10^{22} kg of material and be depleted by over 98% to be the dominant source of the LHB. The criterion of 98% removal is only satisfied in the two cases of 2.5 and 5 lunar masses with Q = 2.5 AU, after 100 Myr and 300 Myr respectively, not counting the obvious case of the 5 lunar mass embryo with Q = 3.3 AU. Thus Fig. 5 suggests that a depletion by 98% is possible for the embryos evolving as in the middle and bottom panels of Fig. 6.

However, depleting only the inner belt would require that the pre-LHB belt had a peculiar structure: for the case where the embryo has 5 lunar masses and Q = 2.5 AU, the inner belt is depleted by 99.7% after 300 Myr while the outer belt is depleted by only 63%. For the case with 2.5 lunar masses the depletion of the inner belt is 98.8% and 56% for the outer belt. At present the innerbelt/outer belt mass ratio is approximately 1:10, and so for the inner belt to have dominated the bombardment the original population ratio needs to have been approximately 13:1 in the first case and approximately 4:1 in the second case. To estimate

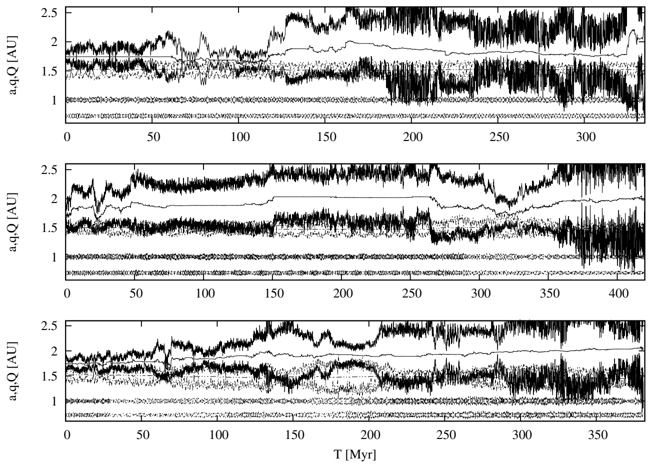

Fig. 6. The evolution of the terrestrial planets which have the embryo crossing the asteroid belt for the longest time. The embryo's mass is 1 lunar mass in the top panel, 2.5 lunar masses in the middle panel and 5 lunar masses in the bottom panel. Each planet is represented by three curves, showing the time-evolution of its orbital q, a and Q. The depletion of the asteroid belt is less than expected from Fig. 5 because these long-lived embryos all have high orbital inclinations.

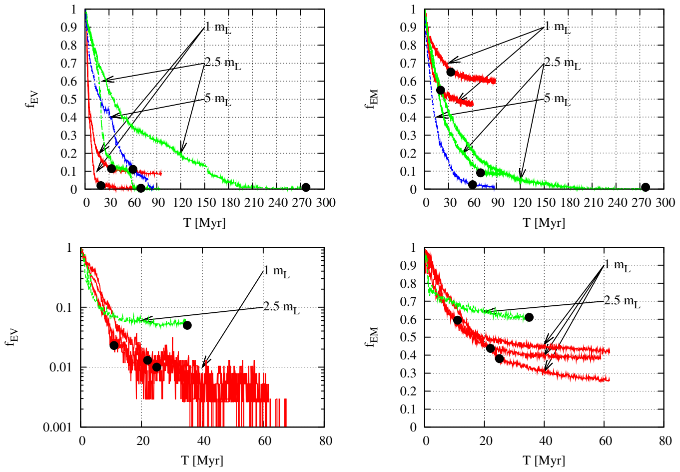

Fig. 7. Decay of the putative belts of small bodies in between terrestrial planets. The left panels refer to the EV belt, the right panels to the EM belt. The top panels are for embryos having an initial $q = 1.2$ AU and the bottom panels for embryos with initial $q = 0.8$ AU. The mass of Planet V is 1 lunar mass (red curves), 2.5 lunar mass (green) and 5 lunar mass (blues). The large bullets in the plots mark the time when the embryo was removed from the simulation, either through ejection or collision. Arrows indicate the mass of the embryo corresponding to each curve.

what the ratio in population density would have been we proceed by giving a crude estimate of the ratio between the volume of phase space of the inner and outer belt. For the inner belt the range in semi-major axis is approximately 0.4 AU and the range in eccentricity is 0.2, while for the outer belt these values are 0.7 AU and 0.3 respectively. Thus the volume occupied by the outer belt is approximately 2.6 times as large as that of the inner belt, and thus the inner belt needed to have been 34 more dense than the outer belt in the first case and approximately 10 times as dense in the second case. Such an extreme density ratio is not supported by the most recent models of primordial asteroid belt sculpting (O'Brien et al. 2007; Walsh et al. 2011). However, it may be compatible with a scenario where the asteroid belt was sculpted by massive planetesimals scattered by Jupiter (Ip 1987; Petit et al. 1999).

3.3. Earth-Venus and Earth-Mars belts

To investigate the destruction of these putative belts, we ran two sets of nine simulations. For the first set, we placed the embryo on a Mars-crossing orbit with $a = 1.4$ AU, $q = 1.2$ AU, $i = 2.3°$ and thus $Q = 1.6$ AU. For the second set the embryo's orbit had $a = 1.44$ AU, $q = 0.8$ AU, $i = 2.4°$ and thus $Q = 2.1$ AU. These orbits were taken from a snapshot of one of the simulations performed without small bodies, described in Sect. 3.1. The masses of the embryo were once again 1, 2.5 and 5 lunar masses. For each value of the mass of the embryo, three simulations have been run with different initial values of the mean anomaly of the embryo.

3.3.1. Decay profiles

The decay of the population of asteroids in the EV and EM belts are depicted in Fig. 7. Some curves end before 100 Myr because in these simulations the embryo was lost quickly – typically within 20 Myr – and since these runs were unsuccessful in liberating enough material in a short enough time, we decided not to continue them. In addition, at the end of approximately half of the simulations, the terrestrial planets were on orbits that are incompatible with the current ones. For example, in one case Mars ended up at 1.4 AU with an eccentricity of 0.13 and in another its semi-major axis was increased to 1.65 AU with an eccentricity of 0.2 after a series of encounters with the Earth. In order to characterise the compatibility of the final terrestrial system with the current one, we decided to adopt the following criterion: the final Angular Momentum Deficit (AMD; Laskar 1997) of the system has to be less than twice the current value of approximately 1.3×10^{-3}, and the final semi-major axes of the terrestrial planets must be within 0.1 AU of their current values. Apart from one 'borderline case' we do not show the decay profiles of any incompatible simulations in Fig. 7 i.e. simulations for which the AMD is too large and/or at least one planet is displaced too far from its current position. In total, we retain nine simulations.

There are some visible trends. When examining the panels on the right, the decay is faster for increasing mass of the embryo, as expected. The lowermost blue curve in the top right panel is for the above-mentioned "borderline successful" simulation, in which Mars ends up onto an orbit with $a \sim 1.45$ AU and $q \sim 1.30$ AU. We report the result of this simulation to illustrate how sensitive the decay curve is on the final orbits of the terrestrial planets. The correlation between embryo's mass and particle decay rate is not as clear in the left panels: in the top panel the red curves (referring to simulations with the lightest embryos) are steeper than the others during the first 10 Myr.

We see in Fig. 7 that, in many cases, the populations of the EV and EM belts continue to decay after the embryo is removed, although at a slower rate. The reason for the continuing decay is that asteroids have been scattered into unstable regions by the embryo before its removal. The simulations where the decay slows down the most tend to be those where the final AMD of the system is comparable to, or less than, the current value. Four out of nine simulations have and AMD value smaller than the current one. In contrast, the simulations where the change in decay rate is less obvious tend to leave us with a terrestrial system whose final AMD is larger than the current value. In simulations where the AMD is larger than the current value the orbits of the terrestrial planets are more dynamically excited than they are now and the meta-stable regions between them are narrower. Consequently, the remaining bodies are removed quicker than they would be if situated in the current terrestrial system.

We noticed that the decay of the EV belt is closely coupled to whether or not the embryo has strong encounters with the Earth i.e. whether or not the Earth was displaced from its original orbit. The reason for this sensitivity is because at the time of the LHB most of the stable asteroids in the EV belt are close to a resonance with the Earth. The typical relative width of these resonances is $\Delta a/a \sim (m_\oplus/M_\odot)^{1/2} \sim 10^{-3}$. Thus, if an encounter with the embryo displaces the Earth by approximately 10^{-3} AU in semi major axis, most of the population becomes unstable and then decays fast. The steepest portions of the decay curves all correspond to these Earth-scattering events.

For the EM belt, however, the radial displacement of the Earth does not have much of an effect on the belt's decay rate. This is because the EM belt contains a region between 1.2 AU and 1.3 AU where the asteroids are very stable and not in mean-motion resonance with either Earth or Mars. Thus, they are not very sensitive to a small radial displacement of the planets.

We found that the decay profiles of the EV and EM belts can be fit rather well with the Kohlrausch law, whose funtional form is given by (Dobrovolskis et al. 2007)

$$\ln f = -\left(\frac{t}{t_0}\right)^{\beta}. \tag{1}$$

When the embryo is still in the system, typically the values of β and t_0 that fit the decay curves are $\beta \sim 0.8$ and $t_0 \sim 4$ Myr for the EV belt, and $\beta \sim 0.9$ and $t_0 \sim 26$ Myr for the EM belt. Once the embryo has been dynamically removed the rate of decay of both populations decreases dramatically as discussed above. For the EV belt and a final AMD of the system similar to or less than the current value the parameters are typically $\beta \sim 0.14$ and $t_0 \sim 10$ Myr while for the EM belt $\beta \sim 0.17$ and $t_0 \sim 26$ Myr. When considering this final decay only, and assuming that the fit is valid for an extrapolation of the decay rate over a timespan 40 times longer than that over which the fit is calibrated, both the EV belt and EM belt decrease by approximately an additional 90% over 4 Gyr. This is probably an overestimate of the total fraction removed: in fact it is unlikely that the decay rate can accelerate again, since the motion of the terrestrial planets is now regular. Instead, it is possible that some stable particles remain in the belt and that, on the long term, the decay curve would reach an asymptotic value corresponding to the fraction of the initial population that survives on stable orbits.

Keeping this in mind, these extrapolations suggest that the fraction of the EM belt that should survive today is between 0.1% and 6%. For the EV belt it is <0.1% for 77% of the simulations where the final configuration of the planets is compatible with the current system, and the remaining fraction is <0.01% in 66% of the compatible final systems.

3.3.2. Which populations of the EV or EM belts could have dominated the LHB?

In order to determine whether or not Planet V can dislodge enough material from the fictitous EV and EM belts to explain the LHB we first compute the average collision probability and impact velocity with the Moon of the asteroids escaping from these belts.

We first investigated how many physical collisions occurred in the simulations. Summing up over all the simulations presented above we record ~3000 impacts on the Earth out of approximately 20 000 asteroids that have been eventually removed either by collision or ejection. This corresponds to a mean collision probability with the Earth of $\langle p_{\rm E}\rangle = 15\%$. The corresponding standard deviation is $\sigma_{\rm p} = 2\%$. The mean Earth impact velocities recorded in the simulations are $\langle v_{\rm iE}\rangle = 16.1$ km s^{-1} with standard deviation $\sigma_{\rm v} = 4.6$ km s^{-1}. Subtracting the gravitational focusing caused by the Earth leaves us with an average encounter velocity "at infinity" (in practice: at the Hill sphere) of $v_0 = 11.5$ km s^{-1}. We can then estimate the average impact velocity on the Moon (which is not included in the simulations) by adding quadratically the escape velocity from the Earth at the lunar distance and the escape velocity from the lunar surface ($v_{\rm fM} = 1.4$ km s^{-1} and $v_{\rm eM} = 2.38$ km s^{-1} respectively). We find that the average impact velocity with the Moon is $\langle v_{\rm iM}\rangle = (v_0^2 + v_{\rm fM}^2 + v_{\rm eM}^2)^{1/2} = 11.8$ km s^{-1}. The average probability of hitting the Moon is then computed as

$$\langle p_{\rm M}\rangle = \langle p_{\rm E}\rangle \left(\frac{R_{\rm M}}{R_{\rm E}}\right)^2 \frac{v_0^2 + v_{\rm fM}^2 + v_{\rm eM}^2}{v_0^2 + v_{\rm eE}^2}, \tag{2}$$

where $v_{\rm eE} = 11.2$ km s^{-1} is the escape velocity of the Earth at its surface and $R_{\rm E}$ and $R_{\rm M}$ are the physical radii of the Earth and Moon respectively. The average impact probability with the Moon amounts to $\sim 0.04\langle p\rangle_{\rm E}$ or approximately 0.6%.

There are approximately ten basins on the Moon which formed during the LHB spike (Wilhelms et al. 1987). Using the impact velocity computed above and using the π-scaling law (Melosh 1989) implies that ten bodies with diameter larger than 15 km impacted the Moon during this epoch. Using the collision probability reported above implies that 2000 bodies of this size should have existed in either the EV or EM belt and be destabilized by the embryo. For comparison, the current asteroid belt now contains about 4000 bodies larger than 15 km in diameter (Jedicke et al. 2002), but in a region approximately 5 times wider in semi-major axis and 2–3 times larger in eccentricity than either of the EV and EM belts. Thus, the EV or EM belt, if they have been the main cause of the LHB, must have been more densely populated than the current asteroid belt.

3.3.3. How severe should the depletion of the EV and EM belts have been to be empty today?

Remember that the EV and EM belts are now empty and that objects larger than 1 km in diameter, if they managed to remain in these belts during the LHB, would still reside there today, because they are big enough not to be removed by the Yarkovsky effect over the last 4 Gyr (see Sect. 2.3).

If either of the considered belts contained 2000 bodies larger than 15 km in diameter to be the cause of the LHB, assuming that the original populations had a cumulative size frequency distribution with a slope of approximately −2, these belts should have contained 450 000 bodies larger than 1 km in diameter. Given that none of these bodies survived until today, the embryo must have been capable to decimate the original population down to a factor of 2×10^{-6}.

Given the uncertainties in the size of the smallest bodies surviving Yarkovsky depletion, the size of the basin-forming projectile, the size distribution slope and the role of collisional removal, in the following we consider "successful" those runs in which the embryo depleted either of the two belts down to a factor 10^{-4} or less. Also given that our simulations started with approximately 1000 objects in each belt, and that we have estimated above that after the embryo is removed a population can still decay by another order of magnitude in the subsequent 3.8 Gyr, we consider "successful" a simulation if either zero or one object remains in either belts *and* the final terrestrial system has an AMD less than twice the current value. We stress that this is most likely an optimistic attitude, so that the fraction of successful simulations mentioned below is probably an upper estimate of the real fraction.

None of our simulations is "successful" for the EM belt. Thus, the EM belt is unlikely to have been a dominant source of the LHB. Moreover we conclude that this belt should not have hosted a substantial population of objects, otherwise it would still be inhabited today.

For the EV belt the prospects are better. As reported at the end of Sect. 3.3.1, a decimation down to 0.01% or less occurs in 66% of the simulations where the final terrestrial planets are compatible with the current system. Thus, in principle, the EV belt can be a sufficent source for the LHB and the rogue embryo could have triggered its global instability.

4. Conclusions

We have investigated, with numerical simulations, a dynamical scenario for the origin of the late heavy bombardment of the terrestrial planets (LHB) originally proposed by Chambers (2007). This scenario argues that a putative fifth terrestrial planet beyond Mars became unstable at the time of the LHB, and subsequently wandered through the asteroid belt or the inner solar system, dislodging asteroids from previously stable populations. We considered several sources of projectiles for the Moon: the main asteroid belt, a putative belt between the orbits of the Earth and Venus (EV belt) and another one between the orbits of the Earth and Mars (EM belt).

We showed that removing enough asteroids to cause the LHB from the entire asteroid belt requires a 5 lunar mass embryo crossing the full asteroid belt belt for approximately 300 Myr on a low-inclination orbit. However, in none of our numerous simulations of the evolution of these planetary systems have we found the fifth planet crossing the full asteroid belt for such a long time. Instead, we discovered that, in a few percent of the cases, the fifth planet can only cross the inner belt (within 2.5 AU) for a comparably long time. Thus, the inner asteroid belt could have been the source of the LHB. However this requires that at the time of the LHB the asteroid belt had a peculiar structure, with an inner part approximately 4–13 times more populated than the outer part, which in turn should have contained approximately twice as many objects as today. This kind of structure has not been witnessed in the most recent models of primordial asteroid belt sculpting (O'Brien et al. 2007; Walsh et al. 2011), but could be compatible with the scenario where the structure of the asteroid belt resulted from the scattering of massive planetesimals by Jupiter (Ip 1987; Petit et al. 1999).

Concerning the putative planetesimal belts among the orbits of the terrestrial planets, we concluded that at least one of these belts needed to contain a number of objects comparable to the current main belt population, but in a much narrower orbital region. No objects exist in these belts today and thus we estimate that the source region of the LHB should have been depleted by at least 99.99% by the effects of the fifth planet. This never happens in our simulations of the evolution of the EM belt, but instead happens quite often for the EV belt. Thus, Chambers' scenario can explain the LHB provided that the EV belt was massively populated, while the EM belt was basically empty. So far no simulation of terrestrial planet formation has ever showed the formation of EV and EM belts with such as strong population dichotomy.

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References

Bottke, W. F., Vokrouhlický, D., Rubincam, D. P., & Nesvorný, D. 2006, AREPS, 34, 157

Bottke, W. F., Levison, H. F., Nesvorný, D., & Dones, L. 2007, Icarus, 190, 203

Chambers, J. E. 2007, Icarus, 189, 386

Chapman, C. R., Cohen, B. A., & Grinspoon, D. H. 2007, Icarus, 189, 233

Dobrovolskis, A. R., Alvarellos, J. L., & Lissauer, J. J. 2007, Icarus, 188, 481

Duncan, M. J., Levison, H. F., & Lee, M. H. 1998, AJ, 116, 2067

Evans, N. W., & Tabachnik, S. 1999, Nature, 399, 41

Gladman, B. J., Migliorini, F., Morbidelli, A., et al. 1997, Science, 277, 197

Hartmann, W. K. 1975, Icarus, 24, 181

Hartmann, W. K., Ryder, G., Dones, L., & Grinspoon, D. 2000, in Origin of the Earth and Moon, ed. R. Canup, & R. Knighter (Tucson, Arizona: Univ. Arizona Press)

Hartmann, W. K., Quantin, C., & Mangold, N. 2007, Icarus, 186, 11

Ip, W.-H. 1987, Gerlands Beitraege zur Geophysik (Sonderheft), 96, 44

Jedicke, R., Larsen, J., & Spahr, T. 2002, in Asteroids III, ed. W. F. Bottke Jr., A. Cellino, P. Paolicchi, & R. P. Binzel (Tucson, USA: University of Arizona Press), 71

Krasinsky, G. A., Pitjeva, E. V., Vasilyev, M. V., & Yagudina, E. I. 2002, Icarus, 158, 98

Kring, D. A., & Cohen, B. A. 2002, JGRE, 107, 5009

Laskar, J. 1997, A&A, 317, 75

Levison, H. F., & Duncan, M. J. 1994, Icarus, 108, 18

Levison, H. F., Dones, L., Chapman, C. R., et al. 2001, Icarus, 151, 286

Melosh, H. J. 1989, Impact cratering: A geologic process (New York: Oxford University Press), Oxford Monographs on Geology and Geophysics, 11

Minton, D. A., & Malhotra, R. 2010, Icarus, 207, 744

Morbidelli, A., & Gladman, B. 1998, M&PS, 33, 999

Morbidelli, A., Jedicke, R., Bottke, W. F., Michel, P., & Tedesco, E. F. 2002, Icarus, 158, 329

Morbidelli, A., Brasser, R., Gomes, R., Levison, H. F., & Tsiganis, K. 2010, AJ, 140, 1391

Neukum, G., & Ivanov, B. A. 1996, MPSS, 31, 97

O'Brien, D. P., Morbidelli, A., & Bottke, W. F. 2007, Icarus, 191, 434

Petit, J.-M., Morbidelli, A., & Valsecchi, G. B. 1999, Icarus, 141, 367

Petit, J.-M., Morbidelli, A., & Chambers, J. E. 2001, Icarus, 153, 338

Ryder, G. 1990, EOSTr, 71, 313

Tera, F., Papanastassiou, D. A., & Wasserburg, G. J. 1974, E&PSL, 22, 1

Walsh, K. J., Morbidelli, A., Raymond, S. N., O'Brien, D. P., & Mandell, A. M. 2011, Nature, 475, 206

Wilhelms, D. L., McCanley J. F., & Trask N. J. 1987, The geologic history of the Moon, US Geological survey professional paper, 1348