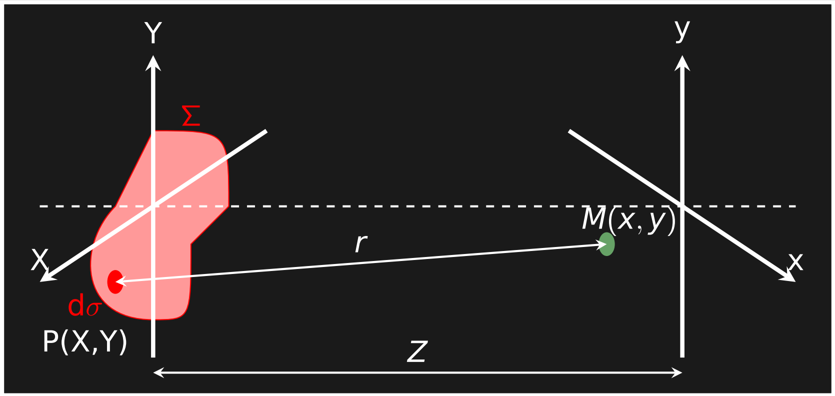
# The high contrast game

Frantz Martinache

September 25, 2017

# Propagating the E-field

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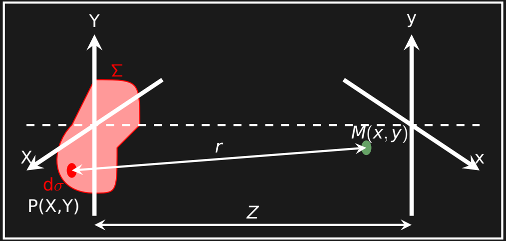
# Fresnel diffraction



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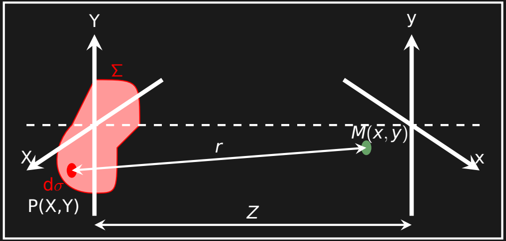

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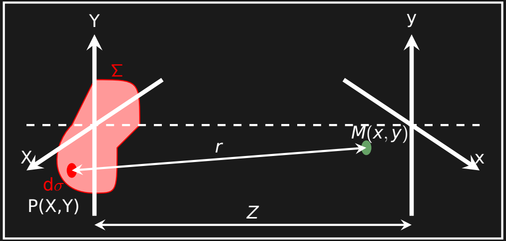


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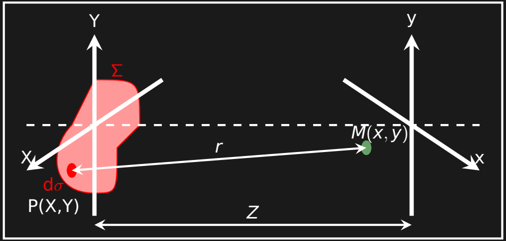


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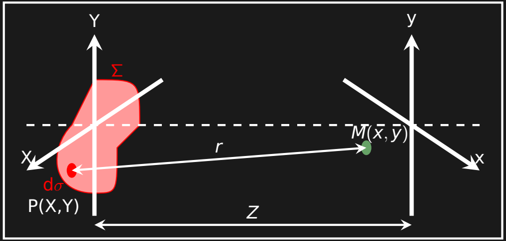


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If Z sufficienctly large:

$$r = \sqrt{Z^2 + (X-x)^2 + (Y-y)^2}$$

$$\approx Z\left(1 + 0.5\left(\frac{X-x}{Z}\right)^2 + 0.5\left(\frac{Y-y}{Z}\right)^2\right)$$

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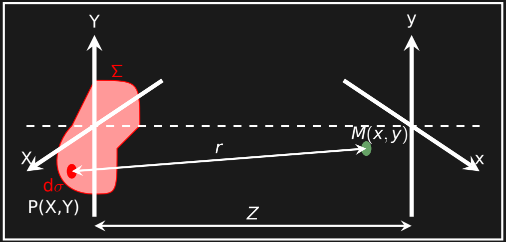


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## Fresnel Transform

$$E(x,y) = \frac{K}{Z} e^{i2\pi Z/\lambda} \iint_{\Sigma} E(X,Y) \exp\left(\frac{i\pi}{\lambda Z}((X-x)^2 + (Y-y)^2)\right)\mathrm{d}\sigma$$

# Far-field diffraction

$$\exp\left(\frac{i\pi}{\lambda Z}(X-x)^2\right) \approx \exp\left(\frac{i\pi}{\lambda Z}x^2\right) \times \exp\left(\frac{-i2\pi}{\lambda Z}xX\right),$$

If, $\frac{X^2}{\lambda Z} << 1$.

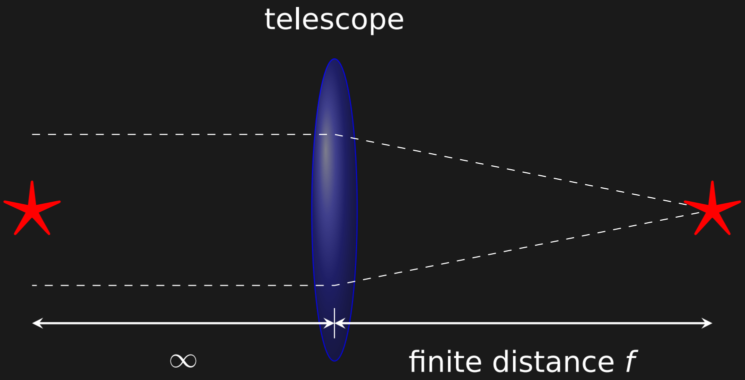
This **approximation** requires the distance Z between the diaphragm and the final screen to be very large compared to the dimension of the aperture.

**Fourier Transform**

$$E(x,y) = K' \iint_\Sigma E(X,Y) \exp\left(-i\frac{2\pi}{\lambda Z}(xX+yY)\right) \mathrm{d}\sigma$$

Compared to the Fresnel Transform, the Fourier Transform is easy to compute.

# Geometric optics to the rescue



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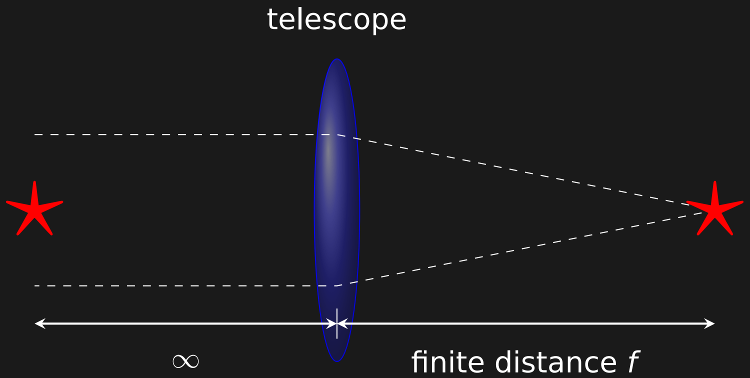


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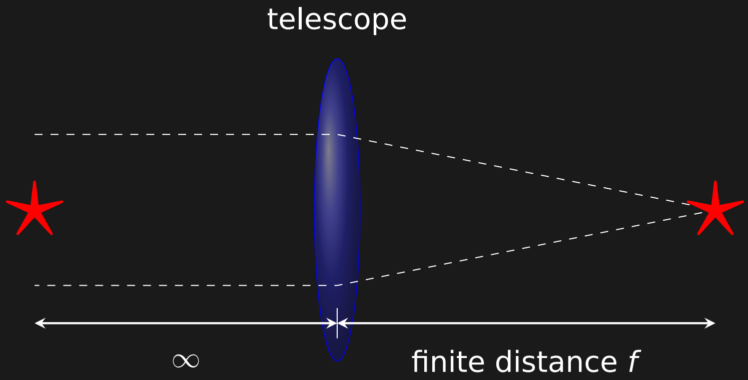


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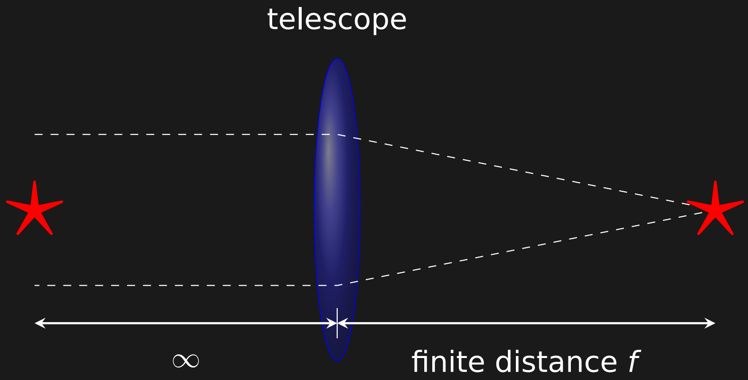


- Fun fact: a powered optics **conjugates** infinity to a finite distance
- **In the focal plane** of a telescope, **Fraunhofer diffraction rules**!
- Between the image and the pupil, Fresnel diffraction must be used.

# The recipe for image formation

Remember the two important coherence properties?

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2. sources are spatially incoherent

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1. we are only able to record the intensity associated to this source: $I_f = |\mathcal{F}(E_p)|^2$
1. if **other sources** are present, **intensity patterns** add-up: $I_{12} = I_1 + I_2$

# Image formation

[HST/NICMOS]

One example:

- Paying only attention to the bright stars in this image
- Each point source produces a similar pattern: spikes + halo
- The size is the same for all (apparent size -> brightness)
- These patterns add-up incoherently (intensities add-up)

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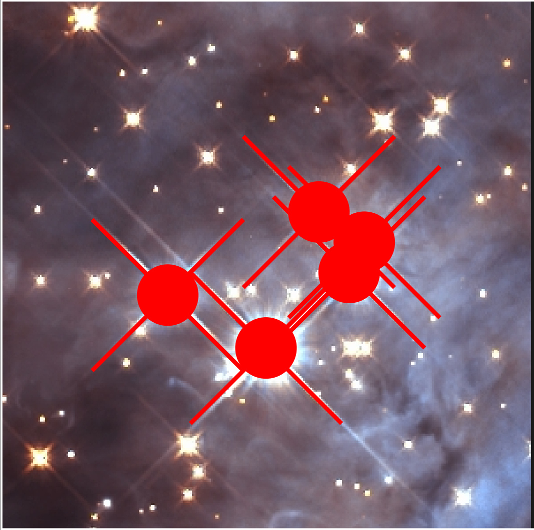

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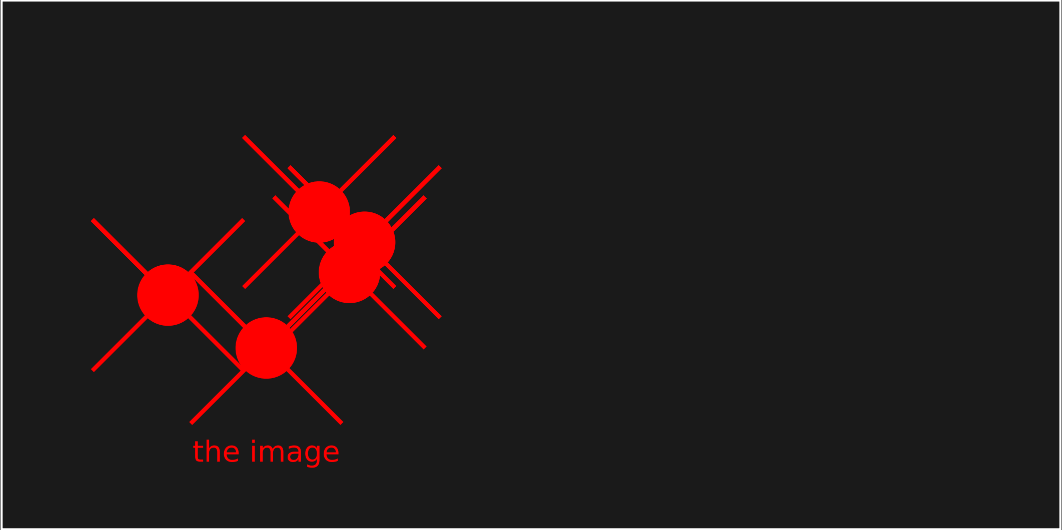

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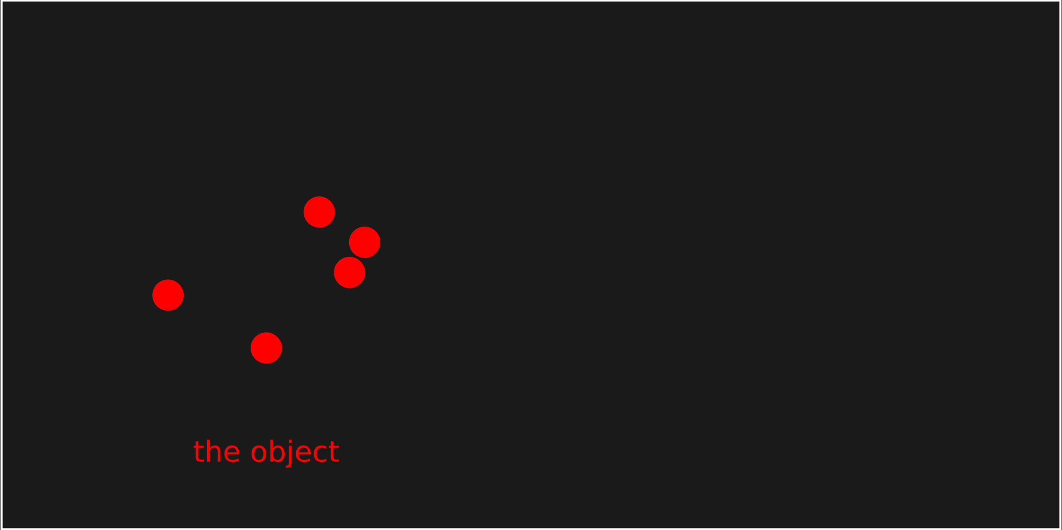

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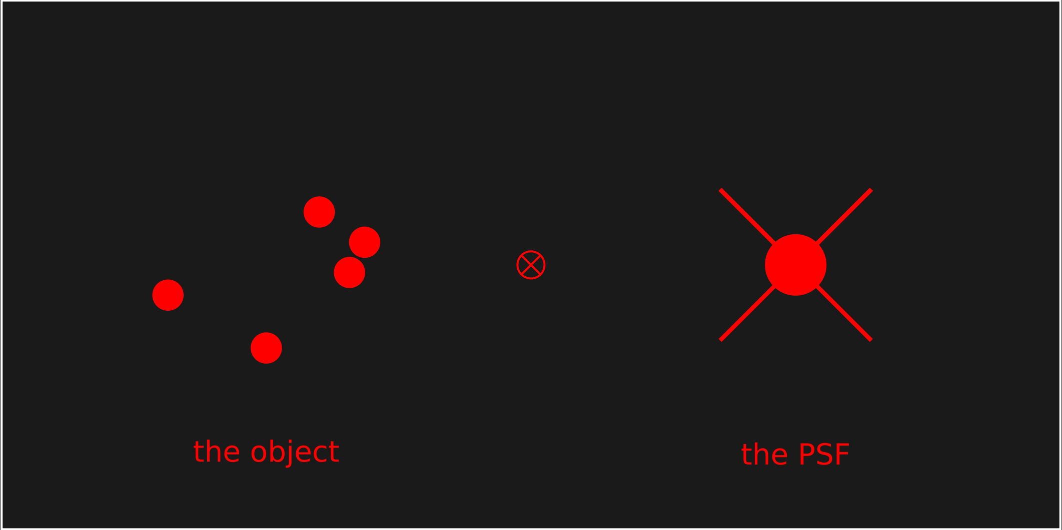
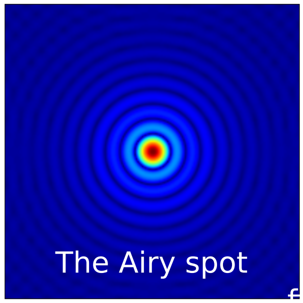


# The circular unobstructed telescope



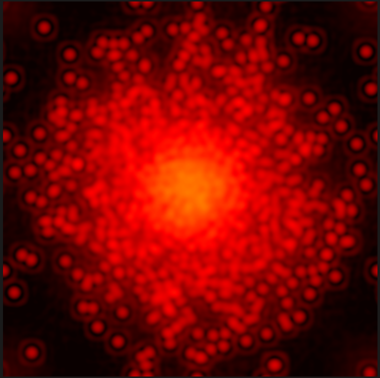

$$\theta = 1.22 \frac{\lambda}{D}$$

$\theta$: first dark ring [radians]

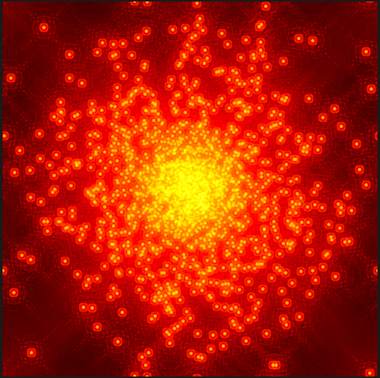
$\lambda$: [meters]

$D$: [meters]

# Angular resolution: aperture size



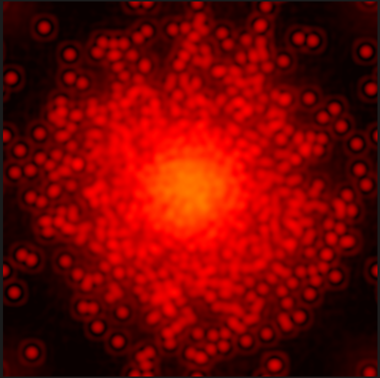
Simulation: 2.5m diameter telescope



Simulation: 8m diameter telescope

**a wider aperture is good for:**

# Angular resolution: aperture size



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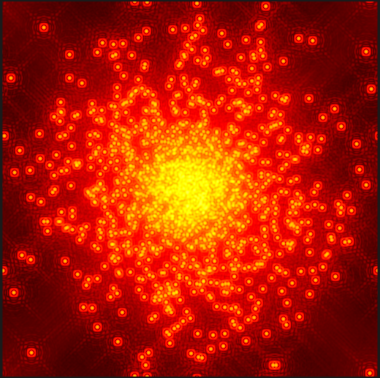
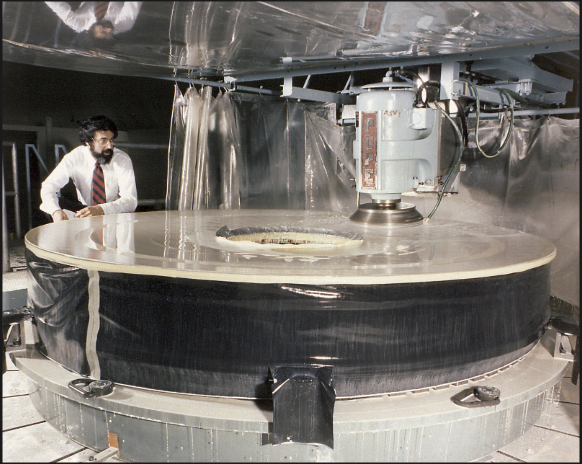


Simulation: 8m diameter telescope

**a wider aperture is good for:**

- a better resolution (wider diameter)
- a higher sensitivity (more collecting area)

# Angular resolution: aperture geometry




[Credit: NASA]

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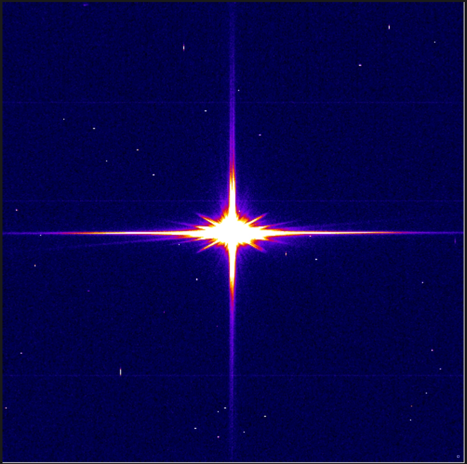
# Angular resolution: aperture geometry


[Credit: ESA]



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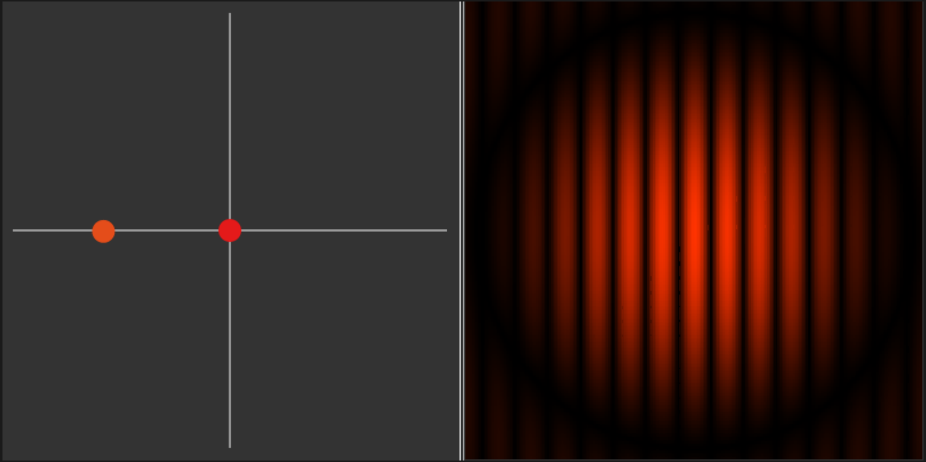


- One **extreme** geometry is the **two-telescope** interferometer
- It provides angular resolution **only** in the direction of the baseline

# Our reference unit: the arc second

- For an 8-meter telescope operating in the near IR: $\lambda$/D $\sim \mu$ radians.
- Instrument plate scales are usually expressed in milli-arc second per pixel.

**convert rad <-> arc-second**

$$\theta\,[''] = \frac{180 \times 3600}{\pi} \times \theta\,[\mathrm{rad}] \simeq 206264.8 \times \theta\,[\mathrm{rad}]$$

**quick trick!**

Estimate your resolution with:

$$\alpha\,[\mathrm{mas}] = 200 \times \frac{\lambda\,[\mu\mathrm{m}]}{D\,[m]}$$

This conversion factor (often rounded to $2\times10^5$) should really be kept in mind.

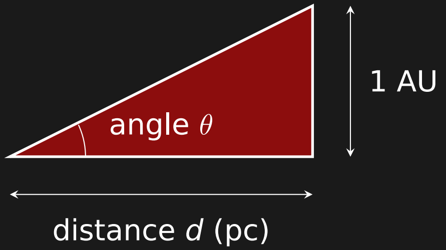
It happens to correspond to the **scaling factor** between phenomena taking place **within the Solar system** and those taking place **outside the Solar system**.

# Side-note: angles and distances

In the Solar system, distances are measured in AU.

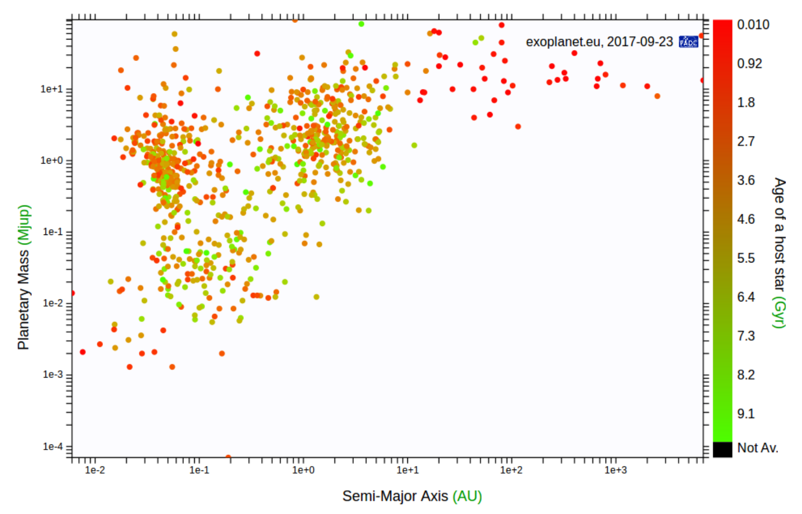
Distances to extrasolar objects are measured in parsecs (pc).

One parsec is the distance at which a projected distance of 1 AU corresponds to an angle of $1''$.



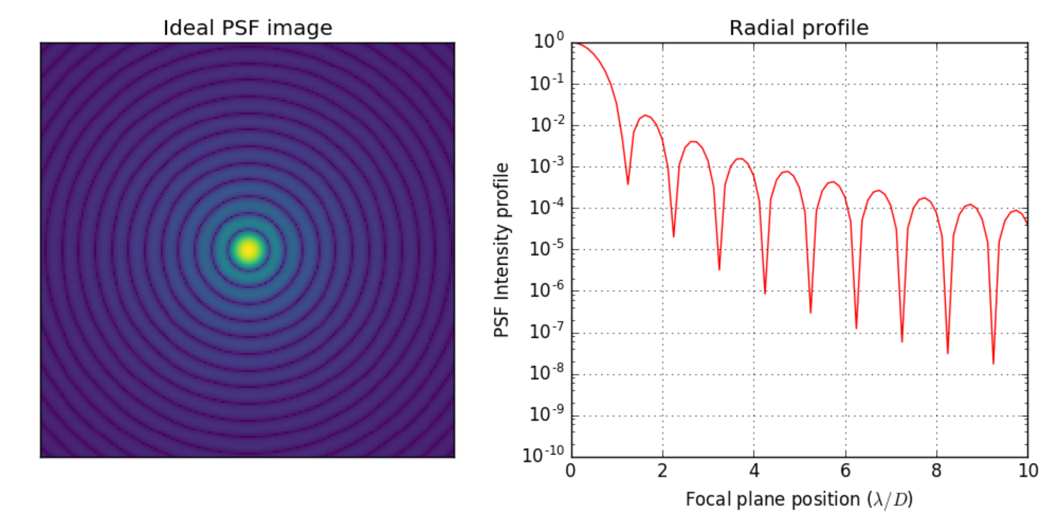

$$\begin{aligned} \tan 1'' \sim 1'' &= 1\,\mathrm{AU}/1\,\mathrm{pc} \\ \theta\,[''] &= 1/d\,[\mathrm{pc}] \\ 1\mathrm{pc} &= 204264.8\,\mathrm{AU} \end{aligned}$$

# Imaging extrasolar planets?




[exoplanet.eu]

# The high-contrast problem



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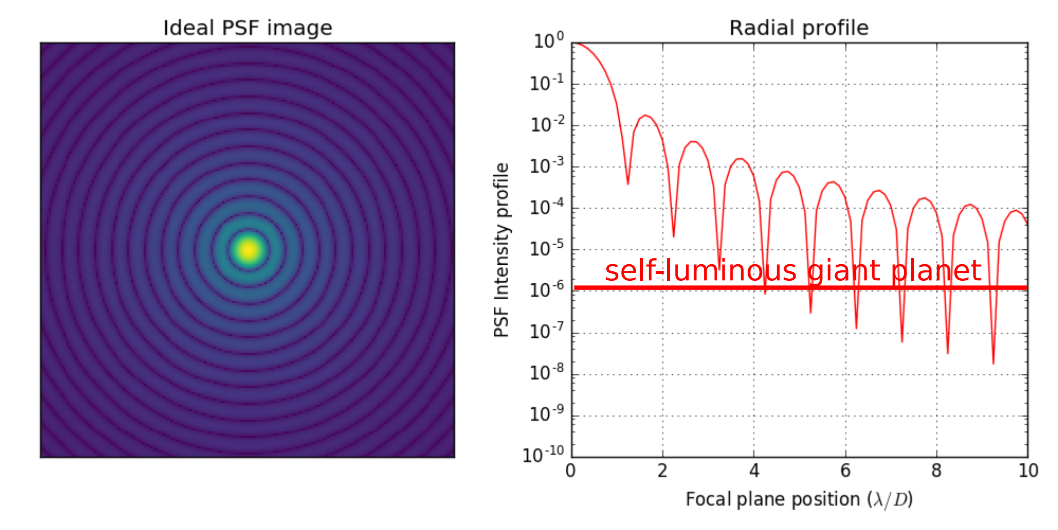

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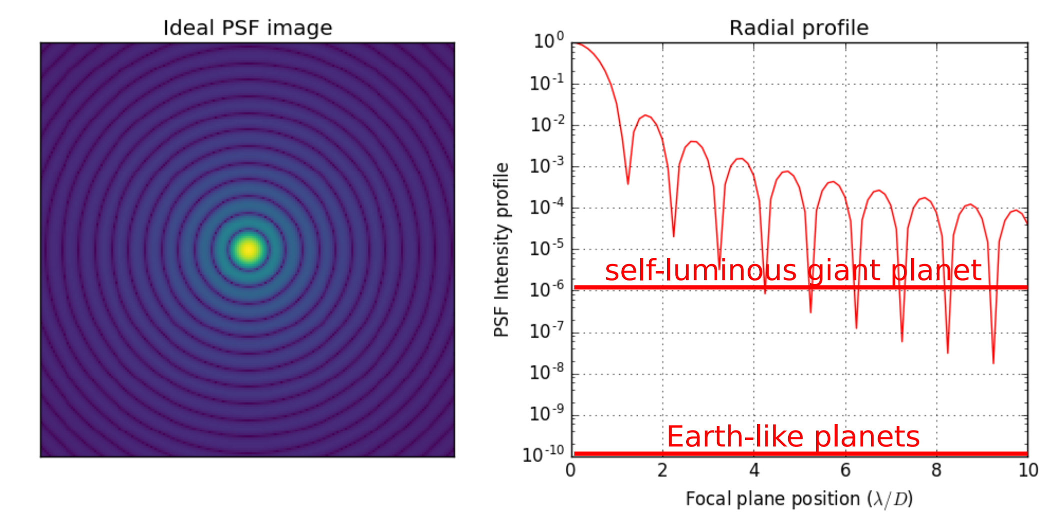

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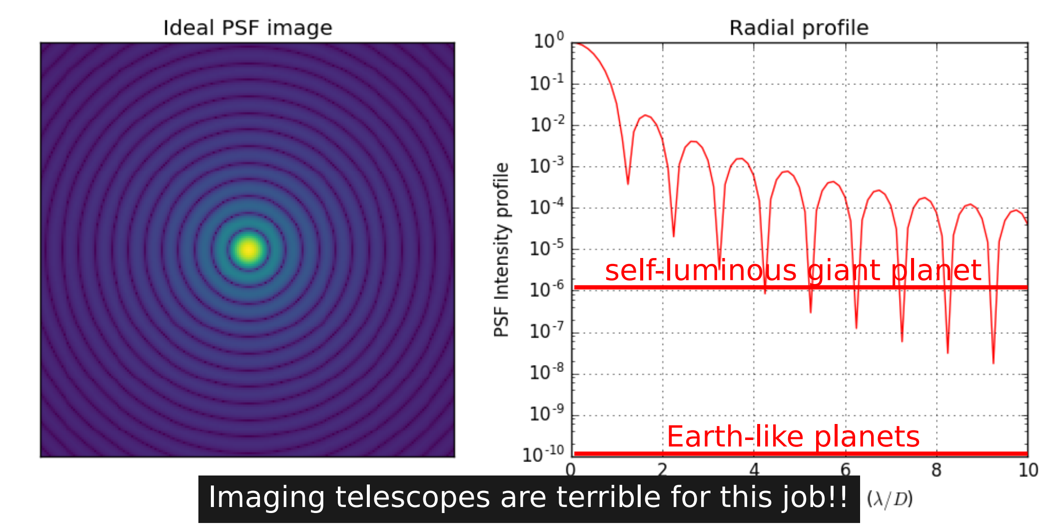
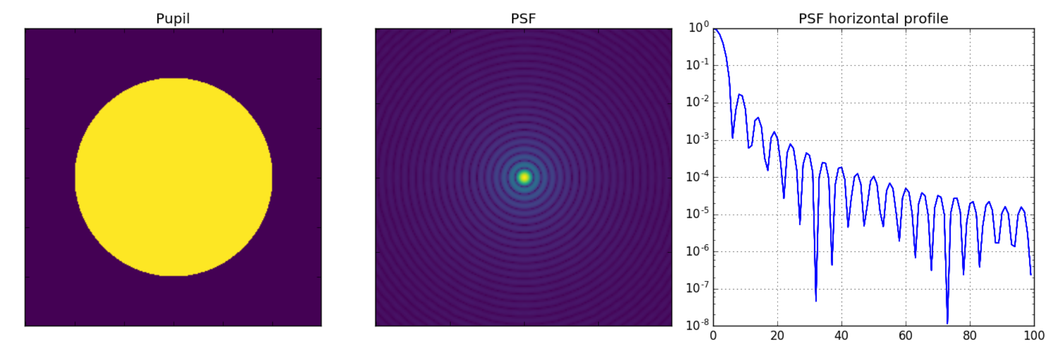



Imaging telescopes are terrible for this job!!

# part 1: apodization

**To apodize: to chop off the foot!**

- the rings (feet) of the PSF are created by the pupil hard edge
- modify the **pupil transmission profile** to attenuate or erase these rings!



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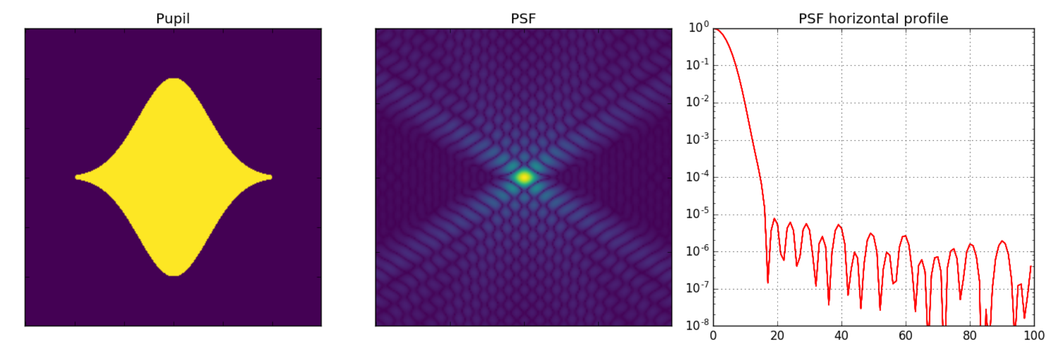

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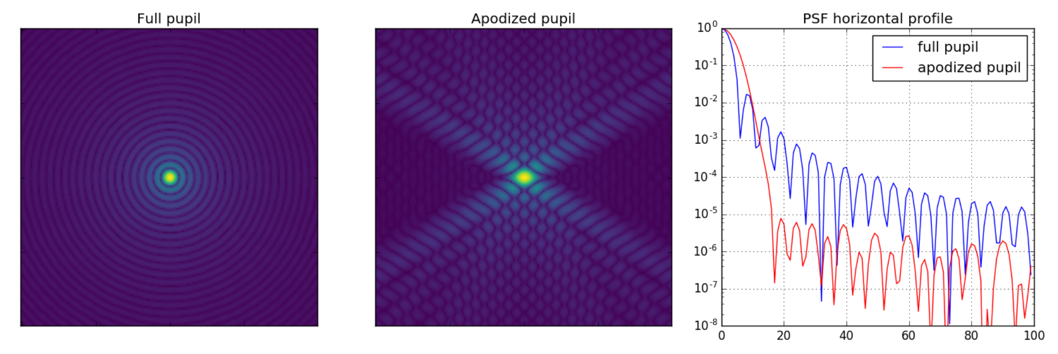

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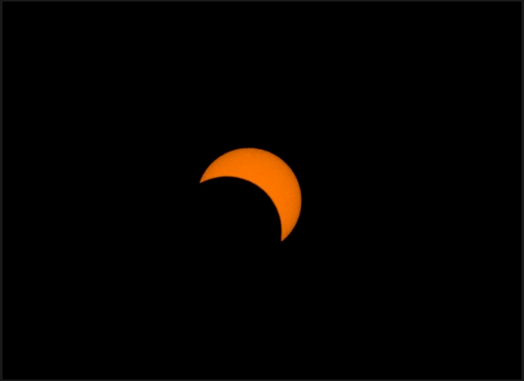

[Image by O. Lardière]

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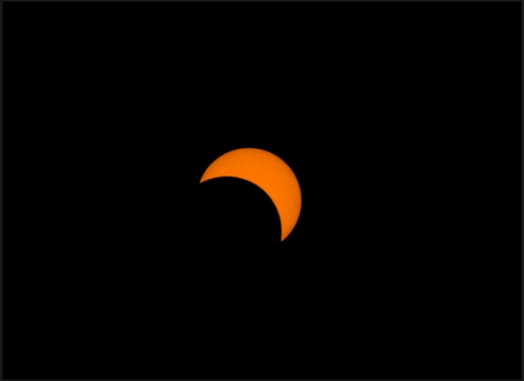

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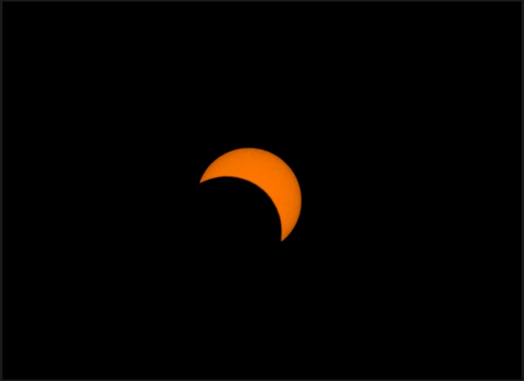

[Image by O. Lardière]

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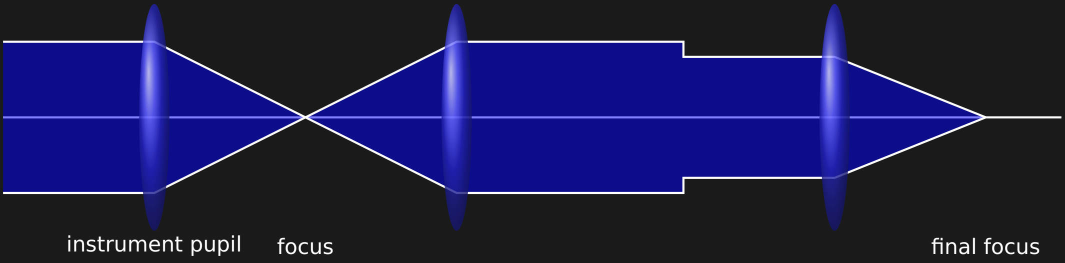
**Optically replicate the eclipse phenomenon**

# possible implementation scheme



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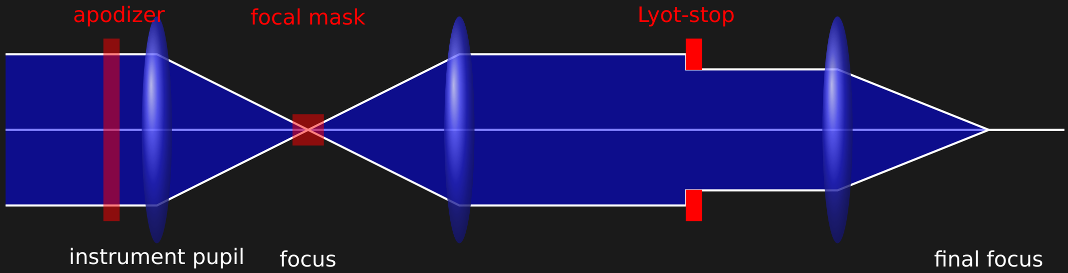

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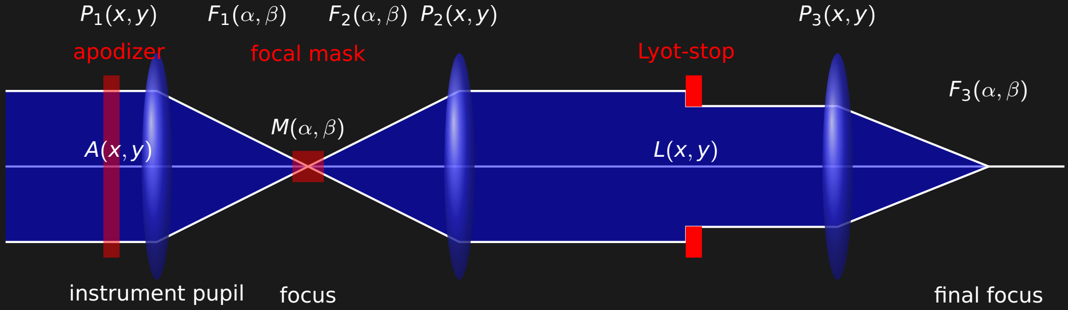

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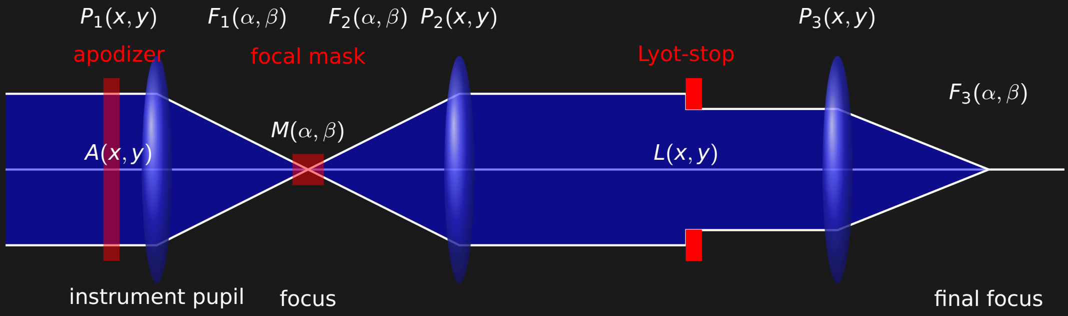
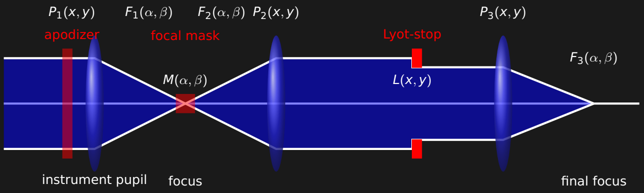



## important remarks

- far-field diffraction applies: Fourier Transform between planes
- elements of the coronagraph interact with the E-field
- final detector records intensity
- because it misses the focal plane mask, off-axis light is mostly unaffected

# the coronagraphic formalism




- pupil coords: $(x, y)$ - image coords: $(\alpha, \beta)$
- apodization function: $A(x, y)$
- focal plane mask function: $M(\alpha, \beta)$
- lyot-stop function: $L(x, y)$
- $\mathcal{F}$ means Fourier Transform

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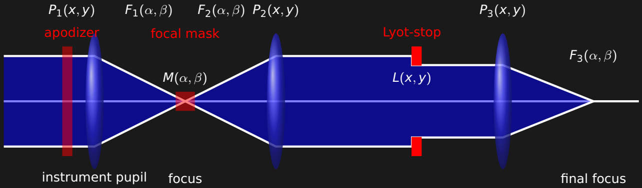


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- $P_1 = A$
- $F_1 = \mathcal{F}(P_1)$
- $F_2 = M \times F_1$
- $P_2 = \mathcal{F}^{-1}(F_2)$
  - $P_2 = \mathcal{F}^{-1}(M \times F_1)$
- $P_3 = P_2 \times L$
- $F_3 = \mathcal{F}(P_3)$
  - $F_3 = \mathcal{F}(P_2) \otimes \mathcal{F}(L)$
  - $F_3 = (F_1 \times M) \otimes \mathcal{F}(L)$
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- $I_3 = |F_3|^2$

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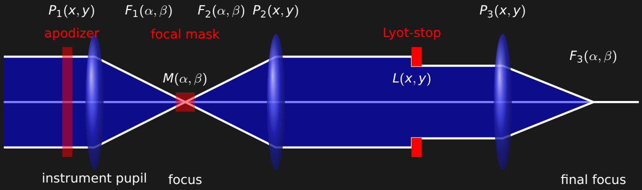


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**Many variants** of coronagraphs exist and it is easy to get carried away looking for the **perfect solution**: **they all share the same weakness!**